

# Introduction to Artificial Intelligence

## Unit # 7

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Spring 2010

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## Popular Machine Learning Techniques

- Classification
  - Classification Trees ✓
  - **Naïve Bayes**
  - Neural Networks
- Clustering
  - K-Means
  - Associative Memory
- **In this course, the focus is on the classification techniques**

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## Conditional Probability Example

	B		~B	
	C	~C	C	~C
A	12	5	9	2
~A	4	8	20	4

- $P(A | B, C) = 12/16$
- $P(A, B | \sim C) = 5 / 19$
- $P(B | \sim A, C) = 4 / 24$

## Conditional Independence

- Two events **A** and **B** are independent if knowing that **A** has happened does not say anything about **B** happening.

$$P(A \cap B) = P(A) P(B)$$

$$P(A | B) = P(A)$$

- Two events **A** and **B** are conditionally independent given a third event **C** precisely if the occurrence or non-occurrence of **A** and **B** are independent events in their conditional probability distribution given **C**.

$$P(A \cap B | C) = P(A | C) P(B | C)$$

$$P(A | B, C) = P(A | C)$$

## Bayes Theorem

- $P(A | B) = \frac{P(B | A) P(A)}{P(B)}$   

$$= \frac{P(B | A) P(A)}{P(B | A)P(A) + P(B | \neg A)P(\neg A)}$$
- $P(A)$  is the prior probability and  $P(A | B)$  is the posterior probability.
- Suppose events  $A_1, A_2, \dots, A_k$  are mutually exclusive and exhaustive; i.e., exactly one of the events must occur. Then for any event  $B$ :  

$$P(A_i | B) = \frac{P(B | A_i) P(A_i)}{\sum P(B | A_i) P(A_i)}$$

## Example I

- According to American Lung Association, 7% of the population has lung cancer. **Of these people having lung disease, 90% are smokers; and of those not having lung disease, 25.3% are smokers.**
- Determine the probability that a randomly selected smoker has lung cancer.

## Example I Solution

- Let L = Lung Cancer, S = Smoker
- Given that
  - $P(L) = 0.07$
  - $P(S | L) = 0.90$        $P(\sim S | L) = 0.10$
  - $P(S | \sim L) = 0.253$        $P(\sim S | \sim L) = 0.747$
- Find probability,  $P(L | S)$

$$P(L | S) = \frac{P(S \cap L)}{P(S)} = \frac{P(S | L)P(L)}{P(S | L)P(L) + P(S | \sim L)P(\sim L)}$$

$$P(L | S) = \frac{0.9 \times 0.07}{0.9 \times 0.07 + 0.253 \times 0.93}$$

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## Example II

- Assume that about 1 in 1000 individuals in a given organization have committed a security violation.
- Assume that the **sensitivity** of a routine screening polygraph is about 85%. That is, the probability that the polygraph report will indicate a concern is about 85% if the individual has committed a security violation.
- Assume the **specificity** of the polygraph is about 80%. That is, if the individual has not committed a security violation, there is about an 80% chance that the polygraph report will not indicate a concern.
- What is the posterior probability that an individual whose polygraph report indicates a concern has committed a security violation?

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## Example II Solution

- Let
  - S = Security Violation Committed,
  - T = Test Positive
- Given that
  - P(S) = 0.001
  - P(T | S) = 0.85 P(~T | S) = 0.15
  - P(T | ~S) = 0.20 P(~T | ~S) = 0.80
- Find probability, P(S | T)

$$P(S|T) = \frac{P(T|S)P(S)}{P(T|S)P(S) + P(T|\sim S)P(\sim S)}$$

$$P(S|T) = \frac{0.85 \times 0.001}{0.85 \times 0.001 + 0.20 \times 0.999}$$

## Naïve Bayes

### Classification: Mammals vs. Non-mammals

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	yes	mammals
python	no	no	no	no	non-mammals
salmon	no	no	yes	no	non-mammals
whale	yes	no	yes	no	mammals
frog	no	no	sometimes	yes	non-mammals
komodo	no	no	no	yes	non-mammals
bat	yes	yes	no	yes	mammals
pigeon	no	yes	no	yes	non-mammals
cat	yes	no	no	yes	mammals
leopard shark	yes	no	yes	no	non-mammals
turtle	no	no	sometimes	yes	non-mammals
penguin	no	no	sometimes	yes	non-mammals
porcupine	yes	no	no	yes	mammals
eel	no	no	yes	no	non-mammals
salamander	no	no	sometimes	yes	non-mammals
gila monster	no	no	no	yes	non-mammals
platypus	no	no	no	yes	mammals
owl	no	yes	no	yes	non-mammals
dolphin	yes	no	yes	no	mammals
eagle	no	yes	no	yes	non-mammals

- Train the model (learn the parameters) using the given data set.
- Apply the learned model on new cases.

Give Birth	Can Fly	Live in Water	Have Legs	Class
yes	no	yes	no	?

## Naïve Bayes Classification: Mammals vs. Non-mammals

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	yes	mammals
python	no	no	no	no	non-mammals
salmon	no	no	yes	no	non-mammals
whale	yes	no	yes	no	mammals
frog	no	no	sometimes	yes	non-mammals
komodo	no	no	no	yes	non-mammals
bat	yes	yes	no	yes	mammals
pigeon	no	yes	no	yes	non-mammals
cat	yes	no	no	yes	mammals
leopard shark	yes	no	yes	no	non-mammals
turtle	no	no	sometimes	yes	non-mammals
penguin	no	no	sometimes	yes	non-mammals
porcupine	yes	no	no	yes	mammals
eel	no	no	yes	no	non-mammals
salamander	no	no	sometimes	yes	non-mammals
gila monster	no	no	no	yes	non-mammals
platypus	no	no	no	yes	mammals
owl	no	yes	no	yes	non-mammals
dolphin	yes	no	yes	no	mammals
eagle	no	yes	no	yes	non-mammals

A: attributes

M: mammals

N: non-mammals

$$P(A | M) = \frac{6}{7} \times \frac{6}{7} \times \frac{2}{7} \times \frac{2}{7} = 0.06$$

$$P(A | N) = \frac{1}{13} \times \frac{10}{13} \times \frac{3}{13} \times \frac{4}{13} = 0.0042$$

$$P(A | M)P(M) = 0.06 \times \frac{7}{20} = 0.021$$

$$P(A | N)P(N) = 0.004 \times \frac{13}{20} = 0.0027$$

Give Birth	Can Fly	Live in Water	Have Legs	Class
yes	no	yes	no	?

$P(A|M)P(M) > P(A|N)P(N)$

=> Mammals

## Example: Play Tennis

Outlook	Temperature	Humidity	Windy	Class
sunny	hot	high	false	N
sunny	hot	high	true	N
overcast	hot	high	false	P
rain	mild	high	false	P
rain	cool	normal	false	P
rain	cool	normal	true	N
overcast	cool	normal	true	P
sunny	mild	high	false	N
sunny	cool	normal	false	P
rain	mild	normal	false	P
sunny	mild	normal	true	P
overcast	mild	high	true	P
overcast	hot	normal	false	P
rain	mild	high	true	N

$$P(P) = 9/14$$

$$P(N) = 5/14$$

Outlook	Temperature	Humidity	Windy	Class
rain	hot	high	false	?

<b>outlook</b>		
$P(\text{sunny} p) = 2/9$	$P(\text{sunny} n) = 3/5$	
$P(\text{overcast} p) = 4/9$	$P(\text{overcast} n) = 0$	
$P(\text{rain} p) = 3/9$	$P(\text{rain} n) = 2/5$	
<b>temperature</b>		
$P(\text{hot} p) = 2/9$	$P(\text{hot} n) = 2/5$	
$P(\text{mild} p) = 4/9$	$P(\text{mild} n) = 2/5$	
$P(\text{cool} p) = 3/9$	$P(\text{cool} n) = 1/5$	
<b>humidity</b>		
$P(\text{high} p) = 3/9$	$P(\text{high} n) = 4/5$	
$P(\text{normal} p) = 6/9$	$P(\text{normal} n) = 2/5$	
<b>windy</b>		
$P(\text{true} p) = 3/9$	$P(\text{true} n) = 3/5$	
$P(\text{false} p) = 6/9$	$P(\text{false} n) = 2/5$	

## Accuracy or Error Rates

- Partition: Training-and-testing
  - use two independent data sets, e.g., training set (2/3), test set(1/3)
  - used for data set with large number of examples

## Metrics for Performance Evaluation

- Focus on the predictive capability of a model
  - Rather than how fast it takes to classify or build models, scalability, etc.
- Confusion Matrix:

		PREDICTED CLASS	
		Class=Yes	Class=No
ACTUAL CLASS	Class=Yes	a	b
	Class=No	c	d

a: TP (true positive)  
 b: FN (false negative)  
 c: FP (false positive)  
 d: TN (true negative)

## Metrics for Performance Evaluation...

		PREDICTED CLASS	
		Class=Yes	Class=No
ACTUAL CLASS	Class=Yes	a (TP)	b (FN)
	Class=No	c (FP)	d (TN)

- Most widely-used metric:

$$\text{Accuracy} = \frac{a + d}{a + b + c + d} = \frac{TP + TN}{TP + TN + FP + FN}$$

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## Limitation of Accuracy

- Consider a 2-class problem
  - Number of Class 0 examples = 9990
  - Number of Class 1 examples = 10
- If model predicts everything to be class 0, accuracy is  $9990/10000 = 99.9\%$ 
  - Accuracy is misleading because model does not detect any class 1 example

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## Cost Matrix

	PREDICTED CLASS		
	$C(i j)$	Class=Yes	Class=No
ACTUAL CLASS	Class=Yes	$C(\text{Yes} \text{Yes})$	$C(\text{No} \text{Yes})$
	Class=No	$C(\text{Yes} \text{No})$	$C(\text{No} \text{No})$

$C(i|j)$ : Cost of misclassifying class j example as class i

## Cost Matrix (Cont'd)

	PREDICTED CLASS		
		True	False
ACTUAL CLASS	True	10	5
	False	1	14

	PREDICTED CLASS		
		True	False
ACTUAL CLASS	True	10	3
	False	3	14

	PREDICTED CLASS		
		True	False
ACTUAL CLASS	True	10	6
	False	0	14

All three confusion matrices have the same accuracy value, i.e., **24 / 30**

What if the cost of misclassification is not the same for both type of errors?

## Cost Matrix (Cont'd)

		PREDICTED CLASS	
		True	False
ACTUAL CLASS	True	10	5x5
	False	1	14

		PREDICTED CLASS	
		True	False
ACTUAL CLASS	True	10	3x5
	False	3	14

		PREDICTED CLASS	
		True	False
ACTUAL CLASS	True	10	6x5
	False	0	14

Suppose the cost of misclassifying True as False is 5 while the cost of misclassifying False as True is 1.

Accuracy values are:  
**24/50, 24/42, 24/54**

## Cost Matrix (Cont'd)

		PREDICTED CLASS	
		True	False
ACTUAL CLASS	True	10	5x4
	False	1	14

		PREDICTED CLASS	
		True	False
ACTUAL CLASS	True	10	3x4
	False	3	14

		PREDICTED CLASS	
		True	False
ACTUAL CLASS	True	10	6x4
	False	0	14

Suppose the cost of misclassifying True as False is **4** while the cost of misclassifying False as True is 1.

Accuracy values are:  
**24/45, 24/39, 24/48**

## Cost-Sensitive Measures

$$\text{Precision (p)} = \frac{a}{a+c}$$

$$\text{Recall (r)} = \frac{a}{a+b}$$

$$\text{F-measure (F)} = \frac{2rp}{r+p} = \frac{2a}{2a+b+c}$$

- | Precision is biased towards C(Yes|Yes) & C(Yes|No)
- | Recall is biased towards C(Yes|Yes) & C(No|Yes)
- | F-measure is biased towards all except C(No|No)

$$\text{Weighted Accuracy} = \frac{w_1a + w_4d}{w_1a + w_2b + w_3c + w_4d}$$

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## Recall and Precision

Actual	Prediction
T	T
T	F
F	T
F	F
F	T
T	T
T	T
T	F
F	T
T	T

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## Recall and Precision

Actual	Prediction
T	T
T	F
F	T
F	F
F	T
T	T
T	T
T	F
F	T
T	T

- Recall = 4 / 6

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## Recall and Precision

Actual	Prediction
T	T
T	F
F	T
F	F
F	T
T	T
T	T
T	F
F	T
T	T

- Recall = 4 / 6
- Precision = 4 / 7
- F-Measure = 8 / 13

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