Introduction to Artificial Intelligence

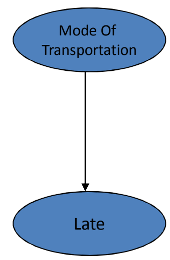
Unit # 11

Bayes Theorem Example

- Suppose that Bob can decide to go to work by one of three modes of transportation, car, bus, or commuter train. Because of high traffic, if he decides to go by car, there is a 50% chance he will be late. If he goes by bus, which has special reserved lanes but is sometimes overcrowded, the probability of being late is only 20%. The commuter train is almost never late, with a probability of only 1%, but is more expensive than the bus.
- Suppose that Bob is late one day, and his boss wishes to estimate the probability that he drove to work that day by car. Since he does not know which mode of transportation Bob usually uses, he gives a prior probability of 1/3 to each of the three possibilities. What is the boss' estimate of the probability that Bob drove to work?

Revised "Bob late to work" Example

- Mode of Transportation: {Car, Not Car (Public Transport)}
- Late: {True, False}
- P(C) = 1/3, P(¬C) = 2/3
- P(L | C) = 0.4, P(L | ¬C) = 0.2



- **P(L) = ?**
- **P(C | L), P(¬C | L)**

Revised "Bob late to work" Example

- One approach is to compute the joint distribution through
  - $P(X_1, X_2, \ldots, X_n = \prod P(X_i \mid parents(X_i))$
  - P(L, C) = P(L | C) P(C) = 0.4 x 0.33
  - P(L, ¬C) = P(L | ¬C) P(¬C) = 0.2 x 0.67
  - P(¬L, C) = P(¬L | C) P(C) = 0.6 x 0.33
  - P(¬L, ¬C) = P(¬L | ¬C) P(¬C) = 0.8 x 0.67
- Now compute P(L), P(C | L)
  - P(L) = P(L, C) + P(L, ¬C)
  - $P(C \mid L) = \frac{P(C, L)}{P(L)}$

Revised "Bob late to work" Example

Or we could use Bayes Theorm based propagation.

$$P(C \mid L) = \frac{P(L \mid C)\,P(C)}{P(L)} = \frac{0.4 \times 0.33}{P(L)}$$

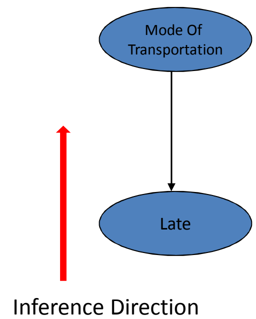
where

$P(L) = P(L \wedge C) + P(L \wedge \neg C)$

$= P(L \mid C)P(C) + P(L \mid \neg C)\,P(\neg C)$

$= 0.4 \times 0.33 + 0.2 \times 0.67 = ?$

Both approaches (on the previous and the current slides) should give the same result

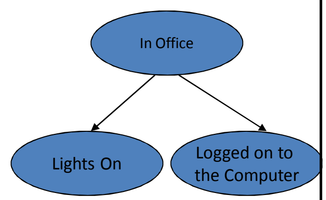


Lecturer's Life Example (Korb & Nicholson)

- Dr. Ann Nicholson spends 60% of her work time in her office. The rest of her work time is spent elsewhere. When Ann is in her office, half the time her light is off (when she is trying to hide from students and get some real work done).
- When she is not in her office, she leaves her light on only 5% of the time.
- 80% of the time she is in her office, Ann is logged onto the computer.
- Because she sometimes logs onto the computer from home, 10% fo the time she is not in her office, she is still logged onto the computer.
- Suppose a student checks Dr. Nicholson's login status and sees that she is logged on. What effect does this have on the student's belief that Dr. Nicholson's light is on?

Lecturer's Life Example (Cont'd)

- In Office (O): {T, F}
- Lights On (L): {T, F}
- Logged on to the Computer (C) : {T, F}
- $P(O) = 0.6$
- $P(L \mid O) = 0.5$, $P(L \mid \neg O) = 0.05$
- $P(C \mid O) = 0.8$, $P(C \mid \neg O) = 0.1$

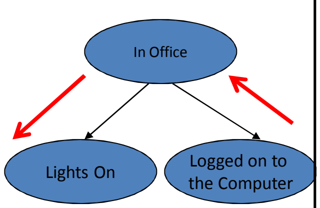


- **$P(L \mid C) = ?$**

Lecturer's Life Example (Cont'd)

- One possibility is to compute the joint distribution and then compute the conditional probability
  - $P(X_1, X_2, \ldots, X_n = \prod P(X_i \mid parents(X_i))$
  - $P(O, L, C) = P(L \mid O)\, P(C \mid O)\, P(O)$
  - $P(O, L, \neg C) = P(L \mid O)\, P(\neg C \mid O)\, P(O)$
  - $P(O, \neg L, C) = P(\neg L \mid O)\, P(C \mid O)\, P(O)$
  - $P(O, \neg L, \neg C) = P(\neg L \mid O)\, P(\neg C \mid O)\, P(O)$
  - $P(\neg O, L, C) = P(L \mid \neg O)\, P(C \mid \neg O)\, P(\neg O)$
  - $P(\neg O, L, \neg C) = P(L \mid \neg O)\, P(\neg C \mid \neg O)\, P(\neg O)$
  - $P(\neg O, \neg L, C) = P(\neg L \mid O)\, P(C \mid \neg O)\, P(\neg O)$
  - $P(\neg O, \neg L, \neg C) = P(\neg L \mid \neg O)\, P(\neg C \mid \neg O)\, P(O)$
- After computing the joint distribution, we can compute $P(L \mid C)$

Inference Direction



Lecturer's Life Example (Cont'd)

- The other approach is to do inference using probability propagation based on Bayes Theorem
- First compute the posterior probability P(O | C) {referred to as P(O*)}
  - $P(O^*) = \dfrac{P(C \mid O)\, P(O)}{P(C)}$
  - And then P(L | C) {referred to as P(L*)}
  - $P(L^*) = P(L \mid O)P(O^*) + P(L \mid \neg O)P(\neg O^*)$